

Parity–Even and Time–Reversal–Odd Neutron Optical Potential in Spinning Matter Induced by Gravitational Torsion

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Recent theoretical work has shown that spin 1/2 particles moving through unpolarized matter which sources torsion fields experience a new type of parity-even and time-reversal-odd optical potential if the matter is spinning in the lab frame. This new type of optical potential can be sought experimentally using the helicity dependence of the total cross sections for longitudinally polarized neutrons moving through a rotating cylindrical target. In combination with recent experimental constraints on short-range P–odd, T–even torsion interactions derived from polarized neutron spin rotation in matter one can derive separate constraints on the time components of scalar and pseudoscalar torsion fields in matter. We estimate the sensitivity achievable in such an experiment and briefly outline some of the potential sources of systematic error to be considered in any future experimental search for this effect.

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INTRODUCTION

Ever since Einstein’s theory of general relativity (GR) successfully proposed an intimate connection between the geometry of spacetime and its matter content, physicists have been encouraged to consider the geometric structure of spacetime as a legitimate subject for scientific study. Among the mathematical quantities that characterize such geometries are curvature and torsion. GR makes essential use of curvature: gravity is interpreted as spacetime curvature and test particle trajectories are geodesics. Spacetime torsion is the other natural geometric quantity that is available to characterize spacetime geometry. Although torsion vanishes in GR, many models which extend GR include various types of nonvanishing torsion sourced by some form of spin density [1]. Yet experimental searches for gravitational torsion are usually specific to a particular torsion model. Even if the coupling of torsion to spin is similar in strength to that of curvature to the energy–momentum tensor, strong spin-density sources which could generate observable effects are difficult to realize. A large fraction of the previous work on gravitational torsion is theory-centric and attempts to argue for specific realizations of torsion in particular theories coming from various mathematical and physical motivations.

By contrast in this work we treat the question of the presence of torsion as an issue to be answered by experiment and make no theoretical assumptions about its possible strength or range. This intellectual perspective favors a qualitatively different experimental strategy which can catch many different torsion possibilities at once. Torsion interactions which violate discrete symme-

tries can be sought with high sensitivity and can benefit from the many powerful techniques of precision measurement which have been developed to search for discrete symmetry violation outside of gravitational physics. We therefore believe that the new possibilities for experimental investigation of torsion along the lines discussed in this Letter are of general interest in the physics community.

Tight model-independent constraints on the size of a very broad set of long-range torsion background fields in spacetime have already been derived from the intellectual perspective we advocate through the appropriate reinterpretation of experiments designed to search for Lorentz and CPT violation [2–4]. This work derived stringent constraints on 19 of the 24 components of a possible ambient torsion field $\mathcal{T}^{\alpha}_{\mu\nu}(x)$ through the coupling of components T^{μ} , A^{μ} and $\mathcal{M}^{\alpha}_{\mu\nu}$ of its irreducible representation [3] to fermions in a general effective Lagrange density with all independent constant-torsion couplings of mass dimensions four and five. Torsion fields which do not extend far from sources would not be seen in the experiments used in these analyses. It is therefore of interest to consider how one might constrain a broad set of possible short-range torsion fields experimentally.

We argue that the most promising experimental observable for the type of broadband torsion searches that we advocate are coherent spin-dependent optical effects in forward scattering. No matter what the range of the torsion fields sourced by fermionic matter, such fields must contribute to the forward scattering amplitude of a spin 1/2 particle by the optical theorem of scattering theory. Given the form of any particular torsion model one could easily evaluate its contribution to the forward

amplitude and therefore make direct contact with experimental bounds. Coherent spin-dependent effects in forward scattering can be sought experimentally with high sensitivity using quantum interference. Torsion interactions which violate discrete symmetries are best to look for as they are relatively insensitive to background effects from other physical processes. Polarized slow neutrons in particular are an excellent choice for such an experimental investigation. Neutrons constitute a massive spin 1/2 probe which can penetrate macroscopic amounts of matter due to their zero electric charge and lack of ionizing interactions with matter, and they can also be used to perform sensitive polarization measurements using various types of interferometric methods.

We therefore focus our attention on polarized neutron optical effects induced by torsion interactions which violate parity and time reversal symmetry in P-odd/T-even, P-even/T-odd, and P-odd/T-odd combinations. Recently the first experimental upper bound has been set [5] on the optical potential from P-odd and T-even short-range torsion fields. The experiment employed transversely polarized slow neutrons that traversed a meter of liquid ^4He . Torsion fields sourced by the protons, neutrons, and electrons in the helium atoms can generate a term in the slow-neutron optical potential proportional to $\vec{\sigma} \cdot \vec{p}$. The $\vec{\sigma} \cdot \vec{p}$ term in the neutron optical potential violates parity and therefore causes a rotation of the plane of polarization of a transversely polarized slow neutron beam about its momentum as it moves through matter [6–9]. The rate of rotation of the neutron's spin about \vec{p} may be characterized by the neutron rotary power $d\phi_{\text{PV}}/dL$, where ϕ_{PV} denotes the angle of rotation and L the distance the neutron has traversed in the sample. For the Lagrange density above in the nonrelativistic limit, $d\phi_{\text{PV}}/dL = 2\zeta$ where ζ is a linear combination of the scalar T_0 and pseudoscalar A_0 torsion components equal to $\zeta = (2m\xi_8^{(5)} - \xi_2^{(4)})T_0 + (2m\xi_9^{(5)} - \xi_4^{(4)})A_0$, where m is the neutron mass and $\xi_8^{(5)}$, $\xi_2^{(4)}$, $\xi_9^{(5)}$ and $\xi_4^{(4)}$ are phenomenological constants defined in [3, 5]. The limit on ζ from this work was $|\zeta| < 9.1 \times 10^{-23} \text{ GeV}$. Later work [10] showed that the limit on ζ from long-range torsion fields using other data could be further improved by 5 orders of magnitude. This measurement constrains a linear combination of possible internal torsion fields of arbitrary range generated by the spin- $\frac{1}{2}$ protons, neutrons, and electrons in the helium. Although future neutron spin rotation experiments could in principle be used to set more stringent torsion constraints, in practice measurements of this type if pushed to higher precision will encounter a background parity-odd spin rotation from the neutron-nucleus weak interaction in the Standard Model. Although this background is calculable in principle, in practice our inability to perform calculations involving the strong interaction for low energy processes makes it impractical to subtract off the Standard Model contribution to parity violation

in this case. We therefore do not anticipate further significant experimental improvements on P-odd neutron-torsion interactions from measurements of this type.

It is interesting to ask whether or not there are other experimental possibilities using slow neutrons which can access short-range torsion effects in matter. In this Letter we point out that the answer to this question is yes if one analyzes neutron optical effects in nonstationary media. The existence of such a term has been demonstrated recently by Ivanov and Wellenzohn [11]. They show that a nonrelativistic spin 1/2 particle moving in a medium rotating with angular velocity $\vec{\omega}$ in the presence of a scalar neutron-torsion coupling can possess a P-even and T-odd term in the neutron-matter optical potential of the form

$$\Phi_{\text{eff}}^{(\text{T-odd})} = -\frac{2}{3} \frac{\mathcal{E}_0}{m} i \vec{\sigma} \cdot \vec{\omega}, \quad (1)$$

where \mathcal{E}_0 is the scalar component of the torsion field, which is equal to $\mathcal{E}_0 = -T_0$ in notations of Kostelecký [3, 12], and $\vec{\omega}$ is the angular velocity of the cylinder rotating around the z -axis of the direction of motion of the neutron beam. Below for closer connection to the notation used by Lehnert *et al.* [5] we set $\mathcal{E}_0 = -T_0$. Note that a measurement of this effect in comparison with the existing data from neutron spin rotation can separate the scalar T_0 torsion component from the pseudoscalar A_0 torsion component. The appearance of this P-even and T-odd torsion-Dirac fermion potential has a geometrical origin [11, 13]. Hadley [14] identified the scalar field equal to the frame dragging term $\frac{d\phi}{dt}$ in the Kerr metric of a spinning massive body as a source for violation of CP-invariance, which is related to violation of T-invariance assuming CPT conservation. In contrast to the P-odd torsion-neutron interaction proportional to $\vec{\sigma} \cdot \vec{p}$ discussed earlier, this P-even and T-odd torsion-neutron potential Eq.(1) is proportional to $\vec{\sigma} \cdot \vec{\omega}$. The contribution of the potential Eq.(1) to the forward amplitude in lowenergy neutron-nucleus scattering for neutrons of momentum p is given by

$$f_{\text{TV}}(0) = -i \frac{1}{3} T_0 R^2 L \omega \varphi_{\text{out}}^\dagger \sigma_z \varphi_{\text{in}}, \quad (2)$$

where R and L are the radius and length of a right circular cylinder rotating around the z -axis. This dependence of the forward amplitude Eq.(2) on the parameters of a rotating cylinder is caused by the existence of the effective T-odd potential Eq.(1) inside the cylinder [11]. φ_{in} and φ_{out} are the column Pauli spinors of the neutron in the initial and final state, respectively. They are eigenfunctions of the operator $\vec{\sigma} \cdot \vec{n}$, i.e. $(\vec{\sigma} \cdot \vec{n})\varphi = \pm\varphi$, where \vec{n} is a unit vector of the neutron position inside the rotating cylinder, characterized by the polar θ and azimuthal ϕ angles, and ± 1 are the neutron spin polarizations. Assuming that in the initial state neutrons are polarized parallel and anti-parallel the x -axis

with the wave functions $\varphi_{\text{in}}^{(\pm)}$ having the following elements $(\pm 1/\sqrt{2}, 1/\sqrt{2})$ and in the final state neutrons are described by the wave functions $\varphi_{\text{out}}^{(\pm)}$ with elements $(\cos(\theta/2), \sin(\theta/2)e^{-i\phi})$ and $(-\sin(\theta/2)e^{+i\phi}, \cos(\theta/2))$, respectively, the T-odd contributions to the s-wave amplitude of scattering of polarized neutrons by nucleus are given by

$$f_{\text{TV}}^{(\pm)}(0) = \mp i \frac{T_0}{3\sqrt{2}} R^2 L \omega \left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} e^{\pm i\phi} \right). \quad (3)$$

According to Stodolsky [15], the contribution of T-odd interaction to the cross section of low energy neutron-nucleus scattering is given by $\sigma_{\text{TV}} = (4\pi/p) \text{Im} \Delta f_{\text{TV}}(0)$, where p is a neutron momentum and $\Delta f_{\text{TV}}(0) = f_{\text{TV}}^{(+)}(0) - f_{\text{TV}}^{(-)}(0)$. Using Eq.(3) for the T-odd contribution to the cross section we obtain the following expression

$$\Delta \sigma_{\text{TV}} = -\frac{8\pi}{3\sqrt{2}} T_0 R^2 L \frac{\omega}{p} \left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \cos \phi \right). \quad (4)$$

To avoid certain systematic effects which can be induced by a spinning cylinder [26, 27], the neutrons should be polarized parallel and anti-parallel the z -axis with the wave functions, which can be obtained from the wave functions $\varphi_{\text{out}}^{(\pm)}$ at $\theta = 0$. Setting $\theta = 0$ we get

$$\Delta \sigma_{\text{TV}} = -\frac{8\pi}{3\sqrt{2}} T_0 R^2 L \frac{\omega}{p}. \quad (5)$$

The experiment would then search for the ω -dependent part of the helicity-dependent component of the polarized neutron cross section difference for neutrons passing through a cylinder rotating with an angular velocity ω and be sensitive to the scalar torsion parameter T_0 .

At first glance the $1/p$ dependence in $\Delta \sigma_{\text{TV}}$ may look strange, since a well-known result from nonrelativistic scattering theory shows that the imaginary part of the forward elastic scattering amplitude tends to zero in the limit $pR \ll 1$ [16]. However this argument does not directly apply to our case of a T-odd torsion optical potential inside matter. A purely imaginary term in the forward scattering amplitude proportional to $1/p$ for the T-odd component of $\Delta \sigma_{\text{TV}}$ is fully consistent with unitarity. At some point in the extreme $p \rightarrow 0$ limit the finite size of the extent of the medium, if nothing else, will eventually come into play and give a finite contribution to the cross section which will prevent $\Delta \sigma_{\text{TV}}$ from diverging.

The P-even and T-odd nature of this observable is quite insensitive to potential backgrounds from known interactions. P-even and T-odd interactions involving Standard Model fields require a violation of C which can be introduced neither at the first generation quark level nor into the gluon self-interaction. Consequently, one needs to consider C violation between quarks of different generations and/or between interacting fields.

P-even and T-odd interactions between identical spin 1/2 fermions vanish identically [17]. Indirect constraints from analyses of radiative corrections to constraints on P-odd and T-odd interaction from electric dipole moment searches [18] are more stringent than the direct experimental constraints. No P-even and T-odd physical effect has ever been observed experimentally. The most sensitive direct experimental upper bounds on P-even and T-odd interactions of the neutron come from an analysis [19] of measurements of charge symmetry breaking in neutron-proton elastic scattering [20–22] and a polarized-neutron transmission-asymmetry experiment using transversely polarized 5.9 MeV neutrons in a nuclear spin-aligned target of holmium [23]. Sensitive experiments to search for P-even and T-odd angular correlations in neutron beta decay [24, 25] have seen no such effects. The observable considered in this work is therefore especially insensitive to possible contamination from other physical effects.

Now we briefly discuss the potential sources of systematic error which might be involved in a measurement of this P-even and T-odd term in the forward scattering amplitude from torsion interactions in the presence of spinning matter proportional to $\vec{\sigma} \cdot \vec{\omega}$. To our knowledge such a measurement has not been considered in the literature. The most worrisome potential systematic effect would be a physical phenomenon which makes an internal magnetic field or a spin polarization in the medium proportional to ω . Such a physical phenomenon exists and is known as the Barnett effect [26, 27], the time-reversed version of the more well-known Einstein-deHass-Alfvén effect [28]. In a rotating medium with a finite magnetic susceptibility, the orbital and spin angular momentum vectors which are responsible for the magnetic susceptibility of non-ferromagnetic media will tend to align with $\vec{\omega}$ and will produce a magnetization in the medium $B = \frac{\omega}{\gamma}$ where γ is the gyromagnetic ratio of the sample. The Barnett effect was observed long ago in ferromagnetic media and has recently been observed experimentally for the first time in a paramagnetic spinning medium [29] in gadolinium, which possesses a very large magnetic susceptibility χ and an internal magnetization $M = \chi B$ of $30nT$ for $\omega = 10^4$ Hz. This effect can be greatly suppressed by using a material with a low magnetic susceptibility. In addition, any unpaired electrons or nucleons in such a rotating medium thereby get polarized and can interact with the polarized neutrons through either the electromagnetic interaction or the spin-dependent strong interaction to generate a spin-dependent term in the total cross section proportional to $\vec{\sigma} \cdot \vec{I}$ where \vec{I} is the relevant rotation-induced nuclear or electron polarization [30–32]. However both of these effects are T-even and therefore the corresponding forward amplitudes are out of phase by $\pi/2$ with respect to the T-odd effect considered in this work.

A wide variety of possible neutron spin rotation effects

in noncentrosymmetric structures that could be induced by rotationally-generated stresses in matter have been estimated theoretically for slow neutrons [33–38] and are very small, since the effects of the chiral electronic structure must be dynamically communicated somehow to the nuclear motion, and often this can only be done through higher-order electromagnetic effects. The passage of slow neutrons through an accelerating material medium produces energy changes in the neutron beam if the boundaries are accelerating according to arguments using the equivalence principle [39–42] and have been recently resolved experimentally using measurements with ultracold neutrons [43–46] but vanish for the case of interest in this work. Various effects involving rotating neutron optical elements [47–49] also do not generate our effect.

Using obvious choices for the material of the spinning cylinder (MgF_2 , silicon) which possess a long neutron mean free path with minimal neutron absorption and small angle scattering and are composed of light nuclei which do not possess low-lying neutron-nucleus resonances, one could achieve a sensitivity to T_0 of 10^{-32} GeV in a practical experiment. Comparing with the existing constraints [5] and [10] on the linear combination of T_0 and A_0 described above, this is of about 10 and 5 orders of magnitude smaller.

The other obvious choice which meets our criteria, namely a P-odd and T-odd torsion-dependent term in the neutron forward scattering amplitude, is also possible in principle [30]. A P-odd and T-odd term in the forward scattering amplitude can be accessed experimentally in polarized neutron optics if the target medium is also polarized. Such an observable can indeed access types of gravitational torsion interactions distinct from the ones discussed above [50]. However the large spin dependence of the neutron-nucleus strong interaction would create severe difficulties for experimental torsion searches of this type. One could realize such a search in practice for neutron-electron torsion couplings by employing special materials which possess nonzero electron polarization and small internal magnetic fields [51].

We have pointed out that the recently-identified P-even and T-odd effects induced by effective low-energy torsion-neutron interactions in rotating media can be sought experimentally by measuring the helicity dependence of the total cross section for neutrons moving through a spinning cylinder. The difference of the cross sections of oppositely polarized neutrons caused by the effective low-energy P-even and T-odd potential Eq.(1), depends linearly on an angular velocity of a rotating cylinder. Such an experiment can access the time component of short-range torsion fields sourced by the atoms in the medium and is sensitive to a different set of torsion fields compared to previous experimental work sensitive to P-odd short-range torsion. We considered a number of potential sources of systematic error in an experiment of this type. We are encouraged that with careful de-

sign such an experiment can be conducted with negligible systematic error. Finally we would like to note that, according to a recent analysis of cosmological constant or dark energy density as induced by torsion fields [52], the measurements of torsion in terrestrial laboratories could shed light on the origin of the Universe creation and dark energy as a relic of the Universe evolution.

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